# Angular and Spectral Sparse Sensing With End-to-End Optimized Nanophotonics

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**Abstract:** We present a method for angle and wavelength sensing for underdetermined imaging systems by performing end-to-end nanophotonic inverse design with a compressed sensing backend. 

Recently, end-to-end nanophotonic optimization has been developed as a novel platform for computational imaging, in which a meta-optical frontend is jointly optimized with a backend reconstruction algorithm [1]. Such a framework has the potential to discover novel imaging systems with improved reconstruction performance, whose design does not necessarily conform to common wisdom. One goal of this work is to incorporate *density-based full-Maxwell topology optimization* into such a framework, treating the permittivity $\varepsilon(\mathbf{r})$ at every position as a degree of freedom. This presents a rich set of forward models, which includes the fascinating possibility of transcending the limitations of paraxial optics to achieve improved resolution. Moreover, by using a physically-informed reconstruction technique, we can investigate the *interplay* of the richer full-Maxwell physics with priors imposed by the reconstruction method. Another goal of this work is to operate in the context of angular and spectral sensing, where a sparsity prior enables compressed sensing reconstruction of under-determined systems (i.e., there are fewer detector pixels than possible angles or wavelengths). Our topology-optimized structures accurately reconstruct angles to within $\lesssim 7\%$ error and wavelengths to within $\lesssim 15\%$ error, demonstrating the value of combining richer physics with compressed sensing techniques in under-determined imaging systems. Our angle-selective structures in particular achieve angular resolution beyond the paraxial regime, and we discuss applications of our technique to the problem of space squeezing.

We use compressed sensing for our physically-informed reconstruction backend. For an imaging problem to be suitable for compressed sensing, the input vector must be sparse in some known basis, and that basis must be incoherent for the sampling basis. We satisfy the first condition by choosing our input angle and wavelength vectors to be sparse. The second condition (incoherence) is realized by inverse-design of the nanophotonic structure. Compressed sensing allows us to make accurate reconstructions in situations with limited detection ability.

Our simulation setup is shown in Fig. 1(a): monochromatic plane waves (characterized by angle of incidence $\theta$ and wavelength $\lambda$) are incident on a structure described by a permittivity profile $\varepsilon(\mathbf{r})$. The intensity profile on the detector is calculated via near-to-far field transformation (integrated over each detector pixel). We perform compressed sensing reconstruction, parametrized by parameter $p$, on the detected power with the fast iterative soft-thresholding algorithm (FISTA) [2] to reconstruct the angle or wavelength. The reconstructed parameters ($\theta^*$, $\lambda^*$) are the outputs of the FISTA reconstruction. The FISTA reconstruction is obtained via the LASSO-regularized inverse scattering problem:

$$\tilde{\mathbf{u}} = \operatorname{argmin}_{\mu} \ \|\mathbf{G}\mu - \mathbf{v}\|^2 + p||\mu||_1, \tag{1}$$

where $\tilde{\mathbf{u}}$ is the reconstructed object, $\mathbf{v}$ the raw image, $\mathbf{G}$ the measurement matrix, and $||.||_1$ the $L_1$-norm. The measurement matrix $\mathbf{G}$ is a function of the geometrical parameters of the optimized metasurface and is optimized to be suitable for compressed sensing.

We use our pipeline, described below and shown in Fig.1(a), to solve two problems – angular (Fig. 1(c)) and spectral sensing (Fig. 1(d)). We select our angles to be evenly spaced by increments of $\Delta\theta = 0.1^\circ$, which puts us firmly in what would normally be the paraxial regime. For spectral sensing, we evenly space our frequencies by increments of $\Delta\nu = 0.01c/\lambda_0$ and compute $\lambda(\nu) = c/\nu$. We choose the number of detector pixels to be less than the number of possible angles or wavelengths, so both problems are *under-determined*. We compute gradients for each iteration through backpropagation and the adjoint method. In the forward pass, near fields are calculated, then computed to the far fields, and reconstructed into the initial vector. In the backward pass, we backpropagate through Lasso regression then compute the structure gradients using the adjoint method. Those gradients are used to perform density-based full-Maxwell topology optimization.

The optimized structures in Fig. 1(c-d) (top) show the map of relative permitivitty $\varepsilon$ in the design region. Our incident light comes from the $+\hat{y}$ direction in relation to the structures (top of the page). $\varepsilon$ within our design

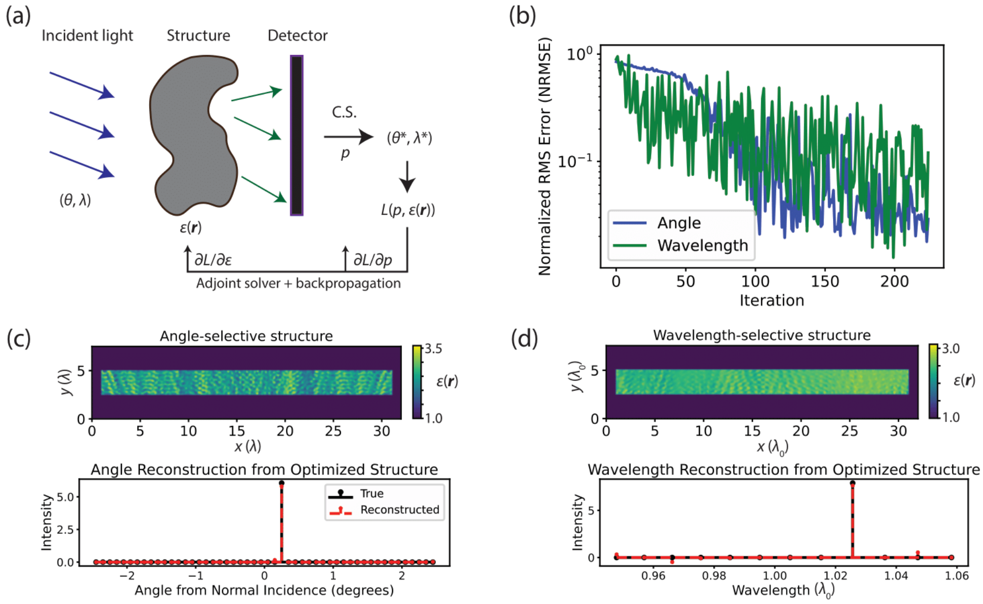


Fig. 1. (a) Schematic of the compressed sensing end-to-end framework for inverse design. We jointly optimize over the structure and the reconstruction parameters. (b) Optimized normalized root-mean-square error (NRMSE) is plotted against the iteration number of optimization steps. (c, d) Resulting structures (top) and an example reconstruction from the angle-detection optimization (bottom) for angle- (c) and wavelength-selective (d) structures. The plot shows the relative permittivity at each position of the design. The x- and y- positions are in wavelengths ($\lambda$) of the incident light for the angle-selective structure or in median wavelengths ($\lambda_0$) for the wavelength-selective structure.

region is constrained to be between $\varepsilon = 1$ and $\varepsilon = 4$. Fig. 1(c-d) (bottom) show example reconstructions of a single input vector for each. For angle sensing, we achieve reconstruction with $\lesssim 7\%$ normalized root-mean-square error (NRMSE); and for spectral sensing, we achieve reconstruction with $\lesssim 15\%$ NRMSE error.

Our compressed sensing reconstruction allows us to achieve accurate reconstructions for under-determined systems. In contrast, most of the work in end-to-end nanophotonics inverse-design so far has been limited to overdetermined problems, which ensure the existence of unique solutions to the reconstruction problem. For underdetermined problems, we do not have such a guarantee, and we rely on our sparsity prior to make up for the missing data. When making measurements is costly, dangerous, or otherwise prohibitive (e.g. medical imaging, radio astronomy), compressed sensing is an attractive way to do more with less.

Our angular reconstruction results are especially significant because they demonstrate the ability of our method to accurately resolve tightly spaced angles, therefore achieving resolution beyond the paraxial regime. Our angular sensing devices can also be understood in the context of the recent interest in nanophotonic space squeezers [3]. In paraxial optics, the minimum distinguishable angle (angular resolution) is dictated by the ratio of the pixel sensor size to the distance between optics and sensor: $\tan\theta = \Delta x_D/d$. Our technique allows us to specify the angular resolution independently of the distance $d$ (equivalent to an optics to sensor distance $d' > d$). Therefore, the proposed structures go beyond paraxial optics and enable sensing of angles at much finer resolutions, not unlike recently-demonstrated space squeezers. In our case, the effective compression factor would be on the order of $\sim 14$, albeit with the additional constraint that the object must be sparse with our technique.

In conclusion, we have demonstrated end-to-end topology-optimized nanophotonic structures for the sparse sensing of angular and spectral information. The approach presented here could in the future be applied to design smaller imaging systems through squeezed space and tackle other imaging problems that have natural sparse priors (as is common in real-world settings).

## References


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